Lepton electric and magnetic dipole moments via lepton flavor violating spin-1 unparticle interactions

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The magnetic dipole moment (MDM) and the electric dipole moment (EDM) of leptons are calculated under the assumption of lepton flavor violation (LFV) induced by spin-1 unparticles with both vector and axial-vector couplings to leptons, including a CP-violating phase. The experimental limits on the muon MDM and LFV process such as the decay $\ell_i^- \to \ell_j^- \ell_k^- \ell_k^+$ are then used to constrain the LFV couplings for particular values of the unparticle operator dimension $d_{\mathcal{U}}$ and the unparticle scale $\Lambda_{\mathcal{U}}$, assuming that LFV transitions between the tau and muon leptons are dominant. It is found that the current experimental constraints favor a scenario with dominance of the vector couplings over the axial-vector couplings. We also obtain estimates for the EDMs of the electron and the muon, which are well below the experimental values.

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I. INTRODUCTION

Scale invariant quantum field theories cannot interpret matter in terms of particles. Motivated by the Banks and Zaks model [1], Georgi [2, 3] conjectured a scenario that, unlike the one posed by the SM and its extensions, introduces scale invariant stuff associated with fractionary anomalous dimension operators. Georgi suggested that a yet unseen scale invariant sector may exist in the high energy theory such that scale invariant stuff may interact weakly with the SM fields. Such a hidden sector may manifest itself at an energy scale $\Lambda_{\mathcal{U}} > 1$ TeV, and since physical particles cannot exist in this sector, the interactions with the SM fields would occur through scale invariant fields known as unparticles. Although a comprehensive study of this class of theories is very complex, it is possible to describe its low energy effects through an effective field theory. This allows one to study the phenomenological effects of unparticle stuff.

The appropriate theoretical framework to describe unparticle physics is the one introduced by Banks and Zaks [1]. The hidden sector is a $\mathcal{BZ}$ sector and the associated fields are described by renormalizable $\mathcal{O}_{\mathcal{BZ}}$ operators. It is assumed that these fields interact with the SM fields through the exchange of heavy particles at a very high energy $\mathcal{M}_{\mathcal{U}}$. Below this energy scale there is nonrenormalizable couplings between the fields of the $\mathcal{BZ}$ sector and the SM fields,[1] which generically can be written as $\mathcal{O}_{SM}\mathcal{O}_{\mathcal{BZ}}/\mathcal{M}_{\mathcal{U}}^{d_{SM}+d_{\mathcal{BZ}}-4}$. Dimensional transmutation is caused by the renormalizable couplings of the $\mathcal{BZ}$ sector at an energy scale $\Lambda_{\mathcal{U}}$ as scale invariance emerges. Below this energy scale, an effective theory can be used to describe the interactions between the SM fields and the $\mathcal{BZ}$ fields, which are associated with unparticles. The effective Lagrangian can be written as [2, 3]:

$$\mathcal{L}_{\mathcal{U}} = C_{\mathcal{O}_{\mathcal{U}}} \frac{\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{\mathcal{M}_{\mathcal{U}}^{d_{SM}+d_{\mathcal{BZ}}-4}} \mathcal{O}_{SM}\mathcal{O}_{\mathcal{U}}, \tag{1}$$

where $C_{\mathcal{O}_{\mathcal{U}}}$ stands for the coupling constant and the operator dimension $d_{\mathcal{U}}$ can be fractionary. From theoretical considerations [4], it has been noted [5, 6] that unitarity is guaranteed in the interval $d_{\mathcal{U}} > 1$. The Lorentz structure of unparticle operators is nontrivial but it can be constructed from the nature of the primary operator $\mathcal{O}_{\mathcal{BZ}}$ and its transmutation. Such a Lorentz structure can be scalar, $\mathcal{O}_{\mathcal{U}}$, vector, $\mathcal{O}_{\mathcal{U}}^{\mu}$, spinor or tensor, $\mathcal{O}_{\mathcal{U}}^{\mu\nu}$. The respective propagators of these unparticles along with their interactions with the SM particles have been studied with detail in [1, 2, 7–10].

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[1] The dimension of the respective operators are $d_{\mathcal{BZ}}$ and d_{SM}

Shortly afterwards the unparticle idea came out, the study of its phenomenology was eagerly addressed [7, 10, 11]. One interesting effect could arise from the interference between unparticle and SM amplitudes, such as could occur in the Drell-Yan process at the Tevatron and the LHC [10, 12]: in particular, the most peculiar effects could be observed in the invariant dilepton invariant mass distribution near the Z-pole [10, 12]. This class of interference effects could also be evident in diphoton production at the LHC [13]. As far as the direct production of unparticles is concerned, it has been studied through mono-photon, $e^-e^+ \to \gamma\mathcal{U}$, and mono-$Z$ production, $e^-e^+ \to Z\mathcal{U}$ [11], whereas the production of an unparticle accompanied by a mono-jet was studied in [14]. Several decays of SM particles into unparticles have been examined: $t \to b\mathcal{U}$ [2], $Z \to \bar{f}f\mathcal{U}$ [10, 11], $H \to \gamma\mathcal{U}$ [15], and $Z \to \gamma\mathcal{U}$ [16]. In addition, other topics on unparticle physics have been studied, such as the possible effects of unparticles on CP violation [17, 18], lepton flavor violation (LFV) [19–21], and lepton electromagnetic properties [22–24].

In order to impose constraints on unparticle parameters, several experimental data have been used. The $e^-e^+ \to \gamma\mathcal{U}$ process was studied to explain $\gamma\bar{\nu}\nu$ production at LEP [11]. It was found that LEP data are consistent with the values $\Lambda_\mathcal{U} = 1.35$ TeV for $d_\mathcal{U} = 2$ and $\Lambda_\mathcal{U} = 660$ TeV for $d_\mathcal{U}$ =1.4. Unparticle constraints have also been obtained from experimental data on cosmology and astrophysics [25–28].

Apart from the tree-level effects, loop induced unparticle effects have studied in the literature [21–23, 29]. The electron magnetic dipole moment via scalar and vector unparticles was obtained in [7, 22]. This study was later extended for the muon magnetic moment due to scalar unparticles with LFV couplings [23], whereas the lepton electric dipole moment via scalar unparticles was studied in [24]. In addition, the calculation of the fermion dipole moments via fermion unparticles was presented in [9]. The study of loop induced decays mediated by unparticles has also been addressed, for instance the decays $l_i \to l_j\gamma$ [21, 23] and $Z \to \bar{l}_i l_j$ [29]. In this work we are interested in calculating the spin-1 unparticle contribution to the magnetic dipole moment (MDM) and the electric dipole moment (EDM) of leptons in the most general case when there is LFV interactions. To our knowledge this calculation has not been presented in the literature.

The rest of the work is organized as follows. In Sec. II we present an overview of unparticle operators. Section III is devoted to the results for the lepton electromagnetic vertex mediated by vector unparticles, while the numerical analysis is presented in Sec. IV. The conclusions and outlook are presented in Sec. V.

II. UNPARTICLES INTERACTIONS WITH THE SM FIELDS

The interactions between the SM particles and unparticles occur through the exchange of heavy fields of mass $\mathcal{M}_\mathcal{U}$. Once those heavy fields are integrated out, the effective Lagrangian that describes the interactions between the SM particles and unparticles is obtained. This effective Lagrangian is composed by a tower of effective operators that can be constructed out of the SM fields by invoking the $SU_L(2) \times U_Y(1)$ gauge symmetry. For instance, the effective interactions of spin-0 and spin-1 unparticles with SM fermions are as follows [1, 2, 7, 8, 10]:

$$\mathcal{L}_{\mathcal{U}_S} = \frac{\lambda_S^{ij}}{\Lambda_\mathcal{U}^{d_\mathcal{U}-1}}\bar{f}_i f_j \mathcal{O}_\mathcal{U} + \frac{\lambda_P^{ij}}{\Lambda_\mathcal{U}^{d_\mathcal{U}-1}}\bar{f}_i \gamma^5 f_j \mathcal{O}_\mathcal{U}, \tag{2}$$

$$\mathcal{L}_{\mathcal{U}_V} = \frac{\lambda_V^{ij}}{\Lambda_\mathcal{U}^{d_\mathcal{U}-1}}\bar{f}_i \gamma_\mu f_j \mathcal{O}_\mathcal{U}^\mu + \frac{\lambda_A^{ij}}{\Lambda_\mathcal{U}^{d_\mathcal{U}-1}}\bar{f}_i \gamma_\mu \gamma^5 f_j \mathcal{O}_\mathcal{U}^\mu. \tag{3}$$

where i and j stand for the family index and $\lambda_J^{ij} = C_{\mathcal{O}_\mathcal{U}} \Lambda_\mathcal{U}^{d_{\mathcal{BZ}}}/\mathcal{M}_\mathcal{U}^{d_{SM}+d_{\mathcal{BZ}}-4}$ stands for the associated coupling constants. These effective operators break scale invariance.

Due to the invariant scale nature of unparticles, their propagators can be constructed by means of unitary cuts and the spectral decomposition formula. Therefore, the scalar unparticle propagator is given by:

$$\Delta_F(p^2) = \frac{A_{d_\mathcal{U}}}{2\sin(d_\mathcal{U}\pi)}(-p^2 - i\epsilon)^{d_\mathcal{U}-2} \tag{4}$$

where the $A_{d_\mathcal{U}}$ function, which is introduced to normalize the spectral density [11], is given as follows:

$$A_{d_\mathcal{U}} = \frac{16\pi^2\sqrt{\pi}}{(2\pi)^{2d_\mathcal{U}}}\frac{\Gamma(d_\mathcal{U}+\frac{1}{2})}{\Gamma(d_\mathcal{U}-1)\Gamma(2d_\mathcal{U})} \tag{5}$$

As far as the vector unparticle is concerned, its propagator is given by

$$\Delta_F^{\mu\nu}(p^2) \;=\; \Delta_F(p^2)\pi^{\mu\nu}(p), \tag{6}$$

where $\pi^{\mu\nu}(p)$ is given explicitly as

$$\pi^{\mu\nu}(p) \;=\; -g^{\mu\nu} + a\frac{p^\mu p^\nu}{p^2}. \tag{7}$$

The form of this expression is due to the spin structure of this class of unparticles [11]. Also $p^2 \neq M^2$, which is a reflect of the unphysical nature of unparticles. When the unparticle field is taken as transverse, it turns out that $p_\mu \pi^{\mu\nu}(p) = 0$, which translates into the condition $a = 1$. However, in the context of a specific conformal invariance, $a = 2(d_{\mathcal{U}} - 2)/(d_{\mathcal{U}} - 1)$ [6]. In the limit $d_{\mathcal{U}} \to 1^+$ the propagator $\Delta_F(p^2)$ turns out to be the propagator of a massless scalar particle, as expected:

$$\lim_{d_{\mathcal{U}} \to 1^+} \Delta_F(p^2) = \frac{1}{p^2}. \tag{8}$$

III. UNPARTICLE CONTRIBUTION TO ELECTRIC AND MAGNETIC DIPOLE MOMENTS OF LEPTONS

Among the best measured particle observables, the muon MDM, a_μ, stand outs: it has been measured with an impressive accuracy of 0.54 ppm. The current world average, which is dominated by the measurements of the E281 collaboration at BNL, is given by [30]

$$a_\mu^{\mathrm{Exp.}} = 116592089\ (63) \times 10^{-11} \quad (0.54\mathrm{ppm}), \tag{9}$$

where the statistical and systematic errors, 0.46 ppm and 0.28 ppm, have been added in quadrature.

The SM theoretical prediction is given by the sum of the QED, electroweak and the hadronic contributions. A recent review of these calculations is given in [31]. While the QED and electroweak contributions to a_μ have been calculated with a great precision, the largest uncertainty arises from the hadronic contribution, which is still under revision. The theoretical SM prediction is

$$a_\mu^{\mathrm{SM}} = 116591834\ (48) \times 10^{-11}. \tag{10}$$

where the $\sigma(e^- e^+ \to \text{hadrons})$ data have been used to calculate the leading-order hadronic vacuum polarization contribution [32]. There is thus a discrepancy between the experimental and theoretical predictions larger than 3.6 standard deviations [33]:

$$\Delta a_\mu = a_\mu^{\mathrm{Exp.}} - a_\mu^{\mathrm{SM}} = 255\,(80) \times 10^{-11}. \tag{11}$$

As long as this disagreement is attributed entirely to new physics [34], the allowed minimal an maximal limits for these contributions, with 95% C.L are $\Delta a_\mu = (255 \mp 1.96 \times 80) \times 10^{-11}$. It may be however that such a discrepancy will reduce to an acceptable level once the hadronic contribution is determined with best accuracy.

Another well studied fermion electromagnetic property is the EDM, d_f, which can provide an excellent probe of new sources of CP-violation. In the SM, the Cabbibo-Kobayashi-Maskawa (CKM) mechanism cannot account for the amount of CP violation required to explain the baryogenesis asymmetry of the universe. It has been long known that the experimental observation of an EDM of fundamental particles will hint to new physics as the SM predictions are very small. For instance, the electron EDM is predicted to be negligibly small as it arises up to the three-loop level via the CKM phase. Other models such as supersymetry, multi Higgs and the left-right symmetric model predict much larger values for d_e [35]. The current experimental limit on the electron EDM with 90% C.L. is [36]:

$$|d_e| \leq 1.6 \times 10^{-27}\ \mathrm{e\,cm}, \tag{12}$$

whereas the experimental limits on the positive and negative muon EDMs with 95% C.L. are [37]:

$$|d_\mu^+| \;\leq\; 2.1 \times 10^{-19}\ \mathrm{e\,cm}, \tag{13}$$

$$|d_\mu^-| \;\leq\; 1.5 \times 10^{-19}\ \mathrm{e\,cm}. \tag{14}$$

We will calculate the contribution to the lepton MDM and EDM induced by a spin-1 unparticle with both vector and axial-vector LFV couplings.

A. Lepton dipole moments via spin-1 unparticle

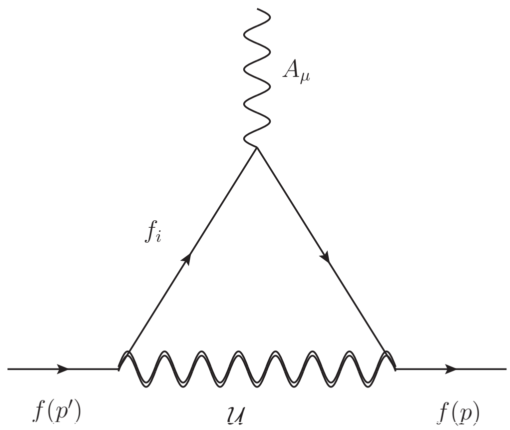

FIG. 1: Feynman diagram for the lepton electromagnetic vertex induced by unparticles.

The Feynman diagram contributing to the electromagnetic vertex is shown in Fig. 1. From Eqs. (2) and (3) we obtain the interactions of the spin-1 unparticle with both vector and axial-vector LFV couplings. We will use Feynman parameters for the calculation and the spectral form for the unparticle propagator will be considered:

$$\Delta_F(p^2) = \frac{A_{d_\mathcal{U}}}{2\pi}\int_0^\infty \frac{dm^2\,(m^2)^{d_\mathcal{U}-2}}{p^2-m^2+i\epsilon}, \tag{15}$$

After the momentum space integration is worked out, we will proceed with the spectral integral.

After some lengthy algebra we arrive at the following results. The MDM of the lepton j due to spin-1 unparticle can be written as

$$a_j^{\mathcal{U}} \;=\; \sum_{J=V,A}\;\sum_{i=e,\mu,\tau} |\lambda_J^{ij}|^2 F_J(m_i, d_\mathcal{U}), \tag{16}$$

where the F_J functions can be written as

$$F_J(m_i, d_\mathcal{U}) = \frac{A_{d_\mathcal{U}}}{16\pi^2\sin(\pi d_\mathcal{U})}\left(\frac{m_i^2}{\Lambda_\mathcal{U}^2}\right)^{d_\mathcal{U}-1} f_J\left(\sqrt{r_i}, d_\mathcal{U}\right), \tag{17}$$

with $\sqrt{r_i} = m_j/m_i$ and

$$\begin{aligned} f_V(z, d_\mathcal{U}) \;&=\; \frac{-z}{2-d_\mathcal{U}}\int_0^1 dx\;(1-x)^{d_\mathcal{U}-1}\,x^{2-d_\mathcal{U}}(1-z^2x)^{d_\mathcal{U}-3}\Big((3(3-x)-4d_\mathcal{U}) + z\,(3(d_\mathcal{U}-1)x + d_\mathcal{U}-3) \\ &+\; z^2\left((1+x)^2 d_\mathcal{U} + (x-5)x - 2\right) - z^3\left(3(d_\mathcal{U}-1)x + d_\mathcal{U}-3\right)x\Big), \end{aligned} \tag{18}$$

also $f_A(z, d_\mathcal{U}) = f_V(-z, d_\mathcal{U})$. We note that there is a flip in the sign of the vector and axial-vector couplings.

For our analysis below, we will also need the contribution to a_μ from a spin-0 unparticle with both scalar and pseudoscalar LFV couplings. We obtain a similar result as that given by Eqs. (16) and (17), with J running over S and P, while the f_S function is

$$f_S(z, d_\mathcal{U}) = -z\int_0^1 dx\;(1-x)^{d_\mathcal{U}}\,x^{1-d_\mathcal{U}}(1-z^2x)^{d_\mathcal{U}-2}\,(1+zx)\,, \tag{19}$$

and $f_P(z, d_\mathcal{U}) = f_S(-z, d_\mathcal{U})$. These results coincide with those presented in [23] and serve as a cross-check for our calculation method.

We would also like to note that in the case of diagonal unparticle couplings, we obtain for the spin-0 and spin-1 unparticle contributions to a_ℓ:

$$a_\ell^{\mathcal{U}} = \frac{A_{d_\mathcal{U}}\Gamma(2-d_\mathcal{U})\Gamma(2d_\mathcal{U}-1)}{16\pi^2\sin(\pi d_\mathcal{U})\Gamma(d_\mathcal{U}+2)}\left(\frac{m_f^2}{\Lambda_\mathcal{U}^2}\right)^{d_\mathcal{U}-1}\left(2(d_\mathcal{U}-2)|\lambda_V^\ell|^2+\frac{4(2-d_\mathcal{U})}{d_\mathcal{U}-1}|\lambda_A^\ell|^2-3|\lambda_S^\ell|^2+(2d_\mathcal{U}-1)|\lambda_P^\ell|^2\right), \quad (20)$$

where $\lambda_J^\ell = \lambda_J^{\ell\ell}$ stands for the diagonal unparticle couplings. This result agrees with the result presented in [22] for the electron MDM.

As far as the lepton EDM is concerned, we will consider the contribution from both spin-0 and spin-1 unparticles. The EDM of fermion j, which can only arise if both λ_V^{ij} and λ_A^{ij} (λ_S^{ij} and λ_P^{ij}) are nonzero and have an imaginary phase, is given by

$$d_j^{\mathcal{U}} = \sum_{(J,K)}\sum_{i=e,\mu,\tau}\mathrm{Im}\left(\lambda_J^{ij}\lambda_K^{*ij}\right)G_{(J,K)}(m_i,d_\mathcal{U}), \quad (21)$$

where (J,K) runs over (V,A) and (S,P). The $G_{(J,K)}$ functions are defined as

$$G_{(J,K)}(m_i,d_\mathcal{U}) = \frac{e\,A_{d_\mathcal{U}}}{32\pi^2\sin(\pi d_\mathcal{U})\,m_i}\left(\frac{m_i^2}{\Lambda_\mathcal{U}^2}\right)^{d_\mathcal{U}-1} g_{(J,K)}\left(\sqrt{r_i},d_\mathcal{U}\right) \quad (22)$$

with

$$\begin{aligned} g_{(V,A)}(z,d_\mathcal{U}) &= \frac{1}{2-d_\mathcal{U}}\int_0^1 dx\;(1-x)^{d_\mathcal{U}-1}x^{2-d_\mathcal{U}}(1-z^2x)^{d_\mathcal{U}-3}\Big(4(1-z^2x)(2-d_\mathcal{U})+(1-3x)\\ &+ z^2\left((x^2-1)d_\mathcal{U}+(x-1)x+2\right)\Big), \end{aligned} \quad (23)$$

$$g_{(S,P)}(z,d_\mathcal{U}) = -\int_0^1 dx\;(1-x)^{d_\mathcal{U}}x^{1-d_\mathcal{U}}(1-z^2x)^{d_\mathcal{U}-2}, \quad (24)$$

In the limit of a heavy internal lepton, $m_j \ll m_i$, the integration can be dealt with and we obtain

$$G_{(V,A)}(m_i,d_\mathcal{U}) = \frac{3(d_\mathcal{U}-2)(d_\mathcal{U}-1)A_{d_\mathcal{U}}}{64\pi\sin^2(\pi d_\mathcal{U})m_i}\left(\frac{m_i^2}{\Lambda_\mathcal{U}^2}\right)^{d_\mathcal{U}-1}, \quad (25)$$

$$G_{(S,P)}(m_i,d_\mathcal{U}) = \frac{(d_\mathcal{U}-1)d_\mathcal{U}A_{d_\mathcal{U}}}{64\pi\sin^2(\pi d_\mathcal{U})m_i}\left(\frac{m_i^2}{\Lambda_\mathcal{U}^2}\right)^{d_\mathcal{U}-1}. \quad (26)$$

These expressions can be useful for the muon and tau loop contributions to the electron EDM and the tau loop contribution to the muon EDM.

IV. NUMERICAL ANALYSIS AND DISCUSSION

Apart from the muon MDM, experimental limits on LFV process are known to be useful to constrain LFV couplings. LFV processes involving the muon are among the most constrained by the experiment. For instance, there are stringent constraints on LFV muon decays: $\mathrm{BR}(\mu\to e\gamma) < 2.4\times10^{-12}$ [38] and $\mathrm{BR}(\mu\to 3e) < 1.0\times10^{-12}$ [39]. Furthermore, the bound on the $\mu\to e\gamma$ rate is expected to be improved by about one order of magnitude by the MEG experiment [40]. However, less stringent constraints exist for LFV τ transitions: $\mathrm{BR}(\tau\to\mu\gamma) < 4.4\times10^{-8}$ [41], $\mathrm{BR}(\tau\to 3e) < 3.6\times10^{-8}$ [42], etc. Below we will analyze the constraints on LFV spin-1 unparticle couplings.

A. Muon anomalous magnetic moment

A comprehensive analysis of the spin-1 unparticle contribution to the muon MDM would require to deal with several free parameters: six coupling constants, the unparticle scale and the unparticle operator dimension. We will take

another approach instead and consider some particular scenarios of LFV together with the hypothesis that there is no large cancellation between different unparticle contributions. First of all, we will assume that there is a hierarchy in the LFV unparticle couplings as it is assumed in other models of LFV, i.e., $|\lambda^{e\mu}| < |\lambda^{e\tau}| < |\lambda^{\mu\tau}| \ll \lambda^{ii}$ for all the unparticle couplings. Since experimental data strongly constrains LFV between the muon and the electron, we will concentrate on the scenario where LFV transitions are dominated by the couplings of spin-1 unparticles with the tau and muon leptons. More specifically, we will focus on the three following scenarios:

A- Vector and axial-vector LFV couplings of similar size.

B- Vector-dominated LFV couplings.

C- Axial-vector-dominated LFV couplings.

To discuss these scenarios we first evaluate numerically each term, $F_J(m_i, d_\mathcal{U})$, of the sum of Eq. (16). The results are shown in Fig. 2 as functions of the dimension $d_\mathcal{U}$ and for $\Lambda_\mathcal{U} = 1$ TeV. We do not show the contributions due to the electron loop as we assumed that the LFV couplings involving the electron are subdominant. While the contributions from vector couplings are positive, the axial-vector contributions are negative. We also note that $F_A(m_i, d_\mathcal{U}) \simeq -F_V(m_i, d_\mathcal{U})$ and so the vector and axial-vector contributions can largely cancel out. This effect is more evident for the contributions of the tau lepton. Furthermore, the contributions from axial-vector couplings are larger in magnitude than the vector contributions, with the largest contribution arising from the muon loop. It means that as long the vector and axial-vector unparticle couplings were of similar order of magnitude (scenario A) the total contribution to the muon MDM could be negative. This is a scenario disfavored by the current experimental data, which requires a positive contribution to $a_\mu^\mathcal{U}$ from new physics. The situation is rather similar for the scalar and pseudoscalar contributions, which for comparison purposes are shown in Fig. 3, although in this case the scalar contributions are slightly larger in magnitude than the axial-vector contributions. If all the unparticle couplings were of similar size, the unparticle contribution to $a_\mu^\mathcal{U}$ could be negative as the axial-vector plus pseudoscalar contributions could be dominant. However, even if all the unparticle couplings were large, the total contribution to a_μ could still be positive by a sort of fine-tuning. We consider the scenario in which the axial-vector couplings are negligible in comparison to the vector couplings (scenario B), such that the total contribution to $a_\mu^\mathcal{U}$ is positive. Below we will analyze the possible constraints on vector unparticle couplings under this assumption.

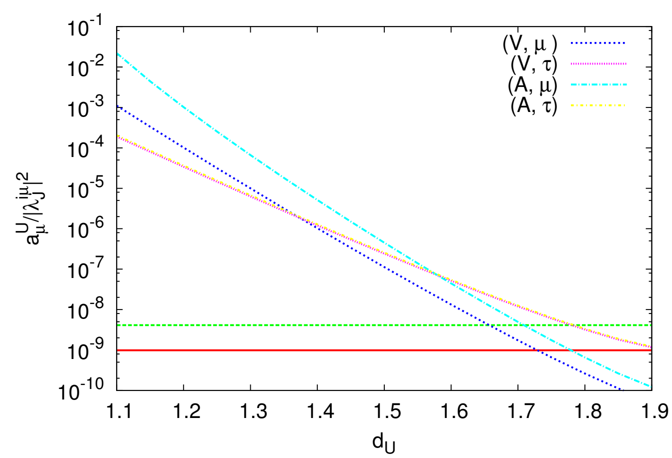

FIG. 2: Partial contributions from the indicated lepton loop to the muon MDM due to vector (V) or axial-vector (A) unparticle couplings as a function of the $d_\mathcal{U}$ dimension and for $\Lambda_\mathcal{U} = 1$ TeV. The absolute values of the (negative) (A) contributions are shown. The horizontal lines are the minimal and maximal allowed limits on new physics contributions to a_μ with 95% C.L. Notice that the contribution from the tau loop are almost indistinguishable.

If the current discrepancy between the theoretical SM prediction and the experimental value of the muon MDM is assumed to be due entirely to unparticle interactions, the allowed limits with 95 % C.L. for this class of contributions are $98.2 \times 10^{-11} \le a_\mu^\mathcal{U} \le 411.8 \times 10^{-11}$. We have found the allowed area on the $|\lambda_V^{\tau\mu}|$ vs $|\lambda_V^{\mu\mu}|$ plane, which is shown in Fig. 4, for several values of $\Lambda_\mathcal{U}$ and $d_\mathcal{U}$ that are consistent with the bounds obtained for the scale $\Lambda_\mathcal{U}$ from mono-photon production at LEP [11] (see Table I). As inferred from Fig. 4, the experimental data allow a magnitude of the vector couplings as large as unity for $d_\mathcal{U} \simeq 2$ and $\Lambda_\mathcal{U} = 1$ TeV, whereas the most strong constraints are obtained for $d_\mathcal{U}$ close to unity. In general the limits on $|\lambda_V^{\mu\tau}|$ are slightly stronger than the limits on $|\lambda_V^{\mu\mu}|$, which is in agreement

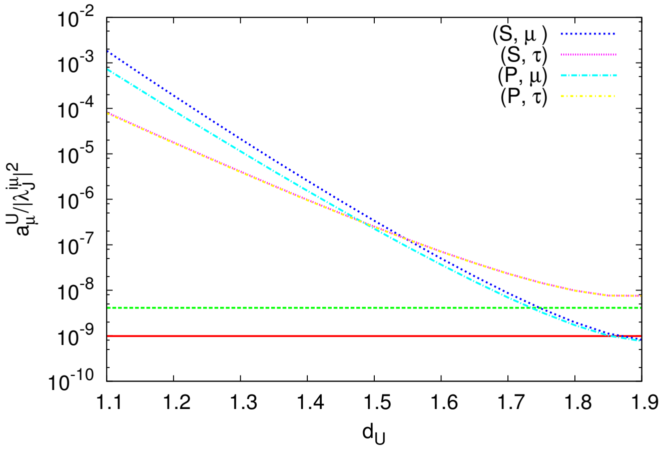

FIG. 3: The same as in Fig. 2, but for the scalar and pseudoscalar contributions to a_μ. The absolute values of the (negative) (P) contributions are shown.

with our assumption. We also note that the bounds on the vector and axial-vector unparticle couplings are of similar order of magnitude than the bounds on the scalar and pseudoscalar unparticle couplings, as shown in Fig. 5. To obtain that plot we have assumed that the dominant contribution to the muon MDM arise from a spin-0 unparticle.

$d_{\mathcal{U}}$	$\Lambda_{\mathcal{U}}$ [TeV]
2.0	1.35
1.8	4
1.6	23
1.4	660

TABLE I: Constraints on $\Lambda_{\mathcal{U}}$ from mono-photon production data at LEP with 95%C.L. and assuming $\lambda_V^{ee} = 1$ [11].

B. Bounds from the decay $\tau \to 3\mu$

It has been long known that LFV couplings can also be constrained from the experimental bounds on the tree-level induced decays $\ell_i^- \to \ell_j^- \ell_k^- \ell_k^+$. We will examine the bounds obtained from the $\tau \to 3\mu$ decay, which involves the $\lambda_V^{\mu\mu}$ and $\lambda_V^{\tau\mu}$ couplings. The calculation for the $\ell_i^- \to \ell_j^- \ell_k^- \ell_k^+$ decay width was already presented in [19] for the scalar unparticle contribution. We have calculated the contribution from vector unparticles and the result is presented in the Appendix A. Let us consider the scenario we are working in, and neglect the axial-vector contributions. With these assumptions we can write for the $\tau \to 3\mu$ branching ratio:

$$\mathrm{BR}(\tau \to 3\mu) \;=\; \frac{m_\tau \tau_\tau}{2^8 \pi^3} \left| \frac{A_{d_{\mathcal{U}}}}{\sin(d\pi)} \right|^2 \left(\frac{m_\tau}{\Lambda_{\mathcal{U}}} \right)^{4(d_{\mathcal{U}}-1)} |\lambda_V^{\mu\mu}|^2 |\lambda_V^{\mu\tau}|^2 \eta_1 \left(\frac{m_\mu}{m_\tau}, d_{\mathcal{U}} \right) \lesssim 10^{-8}, \tag{27}$$

with τ_τ the tau mean life. The right-hand side of the inequality is the experimental constraint [33] and the η_1 function is defined in Appendix A.

Eqs. (16) and (27) can serve to further constrain the allowed values of the coupling constants. We show in Fig. 6 the allowed area obtained after numerical evaluation of Eq. (27) for several values of $\Lambda_{\mathcal{U}}$ and $d_{\mathcal{U}}$. We observe that the $\tau \to 3\mu$ decay constrains considerably the size of the $\lambda_V^{\mu\tau}$ parameter, whose allowed value is extremely small for $d_{\mathcal{U}}$ close to unity, where the $\lambda_V^{\mu\mu}$ allowed values are considerably larger. There is also an allowed area in which the opposite is true ($|\lambda_V^{\mu\tau}| \gg |\lambda_V^{\mu\mu}|$), but we will not consider it as conflicts with our previous assumptions. For comparison purposes, we also have obtained the allowed area on the $|\lambda_S^{\mu\tau}|$ vs $|\lambda_S^{\mu\mu}|$ plane when it is considered that LFV scalar unparticle couplings give the main contributions to the muon MDM and the $\tau \to 3\mu$ decay. The results are shown

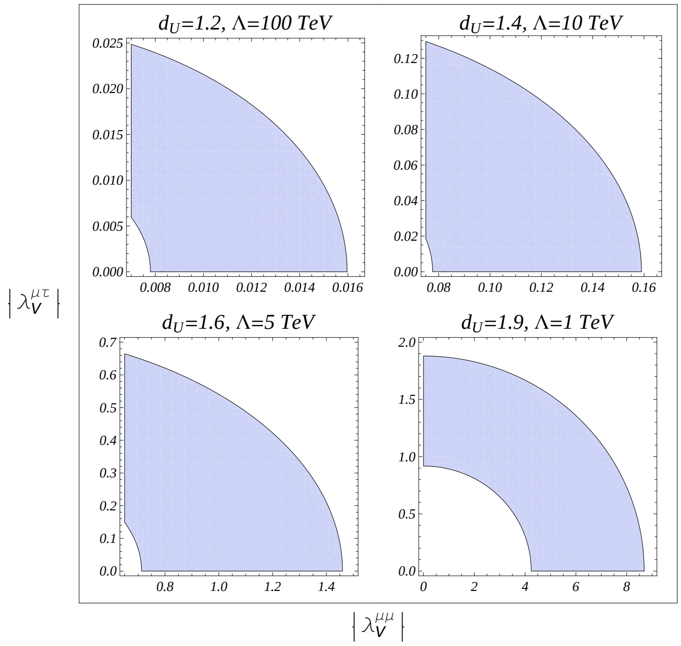

FIG. 4: Allowed area on the $|\lambda_V^{\mu\tau}|$ vs $|\lambda_V^{\mu\mu}|$ plane consistent with the experimental limit on the muon MDM with 95% C.L. We used the indicated values of $\Lambda_{\mathcal{U}}$ and $d_{\mathcal{U}}$.

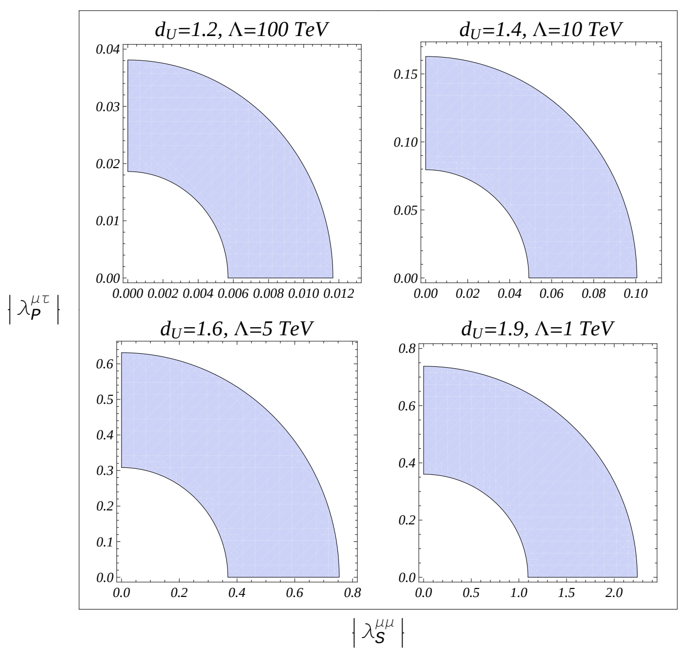

FIG. 5: The same as in Fig. 4, but for the bounds on the $|\lambda_S^{\tau\mu}|$ vs. $|\lambda_P^{\mu\mu}|$ plane.

in Fig. 7 for several values of $\Lambda_{\mathcal{U}}$ and $d_{\mathcal{U}}$. We observe that the allowed size of the scalar unparticle couplings is of similar order of magnitude than for the vector unparticle couplings.

We also analyzed the possible constraints obtained from the $\tau \to \mu\gamma$ decay. It involves the $\lambda_V^{\mu\tau}$, $\lambda_V^{\mu\mu}$ and $\lambda_V^{e\tau}$ couplings. We find that the respective constraints are less stringent than the ones obtained in Fig. 6, so we will not consider this decay mode.

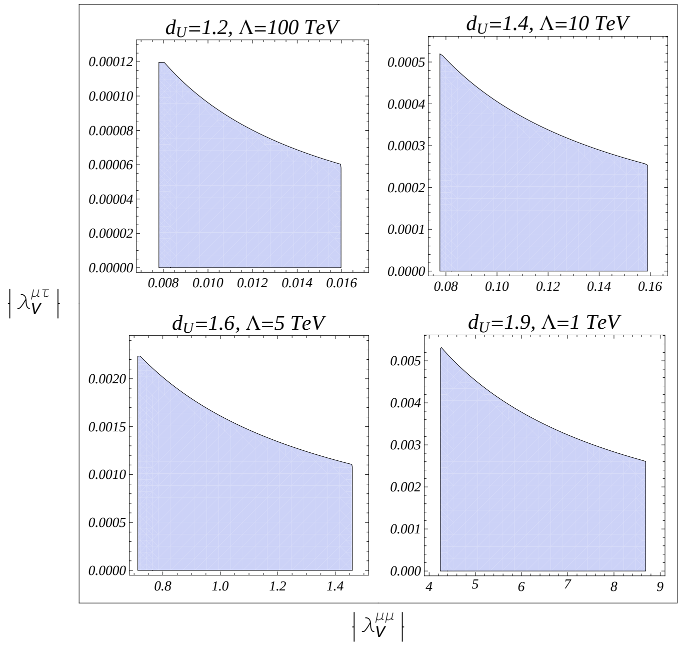

FIG. 6: Allowed area on the $|\lambda_V^{\mu\tau}|$ vs $|\lambda_V^{\mu\mu}|$ plane consistent with the experimental constraints on the muon MDM and the $\tau \to 3\mu$ decay for the values of $\Lambda_{\mathcal{U}}$ and $d_{\mathcal{U}}$ indicated in each plot, when the dominant contribution to these observables arise from the vector unparticle couplings..

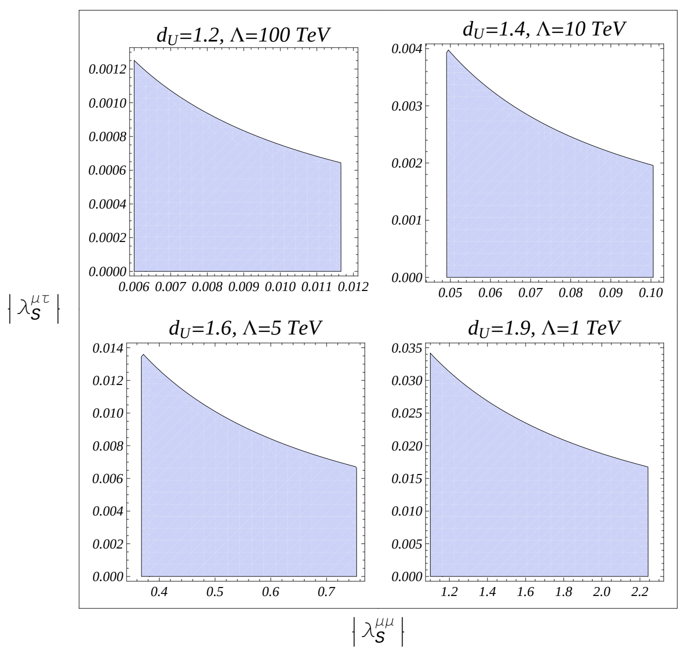

FIG. 7: The same as in Fig. 6, but when the dominant contribution to a_μ and $\tau \to 3\mu$ arise from the scalar unparticle couplings.

C. Electron and muon electric dipole moment

In order to have a nonzero EDM, both kind of unparticle couplings, vector-axial or scalar-pseudoscalar, must be nonzero. Also, an imaginary phase is required. We will try to get an estimate of the order of magnitude for the electron and muon EDMs induced by unparticles. Apart from the magnitude of the $|\lambda_{V,A}^{ij}|$ and $|\lambda_{S,P}^{ij}|$ parameters, the EDM depends on the associated CP-violating phases, so its analysis is even more complicated. The most general

form for the unparticle couplings is $\lambda_J^{ij} = |\lambda_J^{ij}|\exp(-i\theta_J^{ij})$, where $\theta_J^{ij} = -\theta_J^{ji}$ is a CP-violating phase. Therefore $\mathrm{Im}(\lambda_J^{ij}\lambda_K^{*ij}) = |\lambda_J^{ij}||\lambda_K^{ij}|\sin\delta\theta_{(J,K)}^{ij}$, with $\delta\theta_{(J,K)}^{ij} = \theta_J^{ij} - \theta_K^{ij}$ the relative phase between J and K couplings. Depending on the relative sign of the CP violating phases, $\delta\theta_{(V,A)}^{ij}$ and $\delta\theta_{(S,P)}^{ij}$, the partial contributions can add coherently or destructively. We will content ourselves with considering a somewhat restrictive scenario, which will allows us to get an estimate of the order of magnitude of the EDM. First of all, as was the case for the muon MDM, we will assume that there is no large cancellation between different contributions to the EDM, so we can analyze each contribution separately. Also, to be consistent with the previous discussion, we will assume the following hierarchy $|\lambda_{V,S}^{\mu\tau}| \gg |\lambda_{V,S}^{\mu e}|$ and $|\lambda_{V,S}^{e\tau}| \gg |\lambda_{V,S}^{e\mu}|$. It means that the EDM would be entirely driven by the tau loop contributions. In view of this scenario, we numerically evaluate the tau loop contributions to the $G_{(J,K)}$ functions of Eq. (21). The results are shown in Fig. 8. We observe that the contribution from the tau loop to the electron and muon EDMs are indistinguishable, which means that the EDM is entirely controlled by the magnitude of the couplings constants and the imaginary phases. For this calculation we used the approximate formulas given by Eqs. (25) and (26), and made a cross-check with the exact calculation obtained by numerical integration of Eqs. (23) and (24).

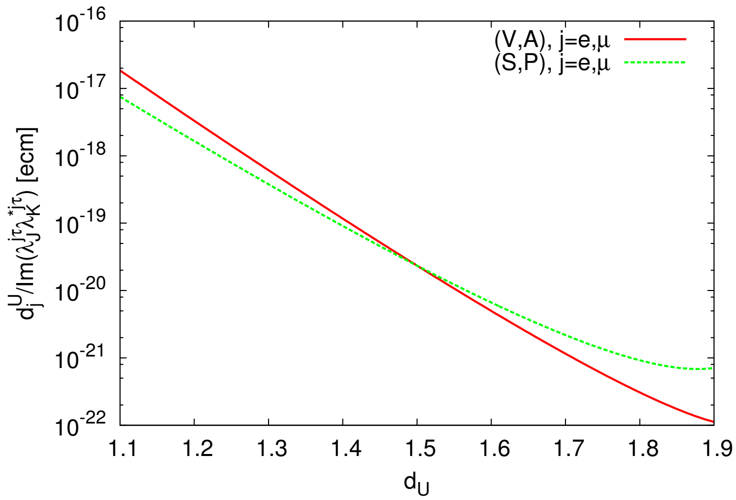

FIG. 8: Partial contribution from the tau lepton loop to the electron and muon EDM due to vector-axial (V, A) and scalar-pseudoscalar (S, P) unparticle couplings as a function of the $d_{\mathcal{U}}$ dimension and for $\Lambda_{\mathcal{U}} = 1$ TeV. The absolute value of the (negative) (V, A) contributions is shown. The curves for the tau contribution to the electron and muon EDMs are indistinguishable.

We now consider the constraints on the coupling constants obtained above to estimate the muon EDM in our working scenario. Both (V, A) and (S, P) contributions are of the order of 10^{-17} e-cm for $d_{\mathcal{U}} \simeq 1.1$ and 10^{-21} e-cm for $d_{\mathcal{U}} \simeq 1.6$. We will consider the case in which the (V, A) coupling prevails over the (S, P) couplings. The allowed magnitude of the $|\lambda_V^{\mu\tau}|$ parameter is of the order of 10^{-5} for $d_{\mathcal{U}} = 1.1$ and 10^{-3} for $d_{\mathcal{U}} = 1.6$. A good assumption for the $|\lambda_A^{\mu\tau}|$ coupling is that its magnitude is about one or two orders of magnitude below $|\lambda_V^{\mu\tau}|$. This would not spoil the bounds we obtained above. Therefore the muon EDM is about $|d_\mu^{\mathcal{U}}| \simeq |\sin\delta_{(V,A)}^{\mu\tau}| \times 10^{-30}$ e-cm for $d_{\mathcal{U}} \simeq 1.1$, whereas $|d_\mu^{\mathcal{U}}| \simeq |\sin\delta_{(V,A)}^{\mu\tau}| \times 10^{-29}$ e-cm for $d_{\mathcal{U}} \simeq 1.6$. These results are well below from the experimental limit on the muon EDM as the imaginary phase is expected to be very small. The magnitude of the coupling constants for LFV transitions involving the electron are expected to be more suppressed as long as the hierarchy discussed above is respected. Then we can estimate that the electron EDM induced by vector unparticles is well below the 10^{-30} e-cm level and far from the experimental measurement. Moreover, since the constraints on LFV spin-0 and spin-1 unparticle couplings are similar, the contributions to the muon and electron EDMs from both spin-0 and spin-1 unparticles are expected to be of similar size. Even if all different unparticle contributions add coherently, it is hard to expect a large size of the electron and muon EDMs due to unparticles.

V. CONCLUDING REMARKS

The lepton magnetic and electric dipole moments were calculated in the framework of unparticle physics assuming interactions mainly driven by a spin-1 unparticle with both vector and axial-vector couplings to leptons. Analytical

expressions were found that agree with previous calculations for the muon MDM in the limit of flavor conserving couplings.

The muon MDM was numerically evaluated to obtain bounds on the coupling constants from the experimental measurements assuming that LFV is mainly dominated by LFV couplings involving the muon and tau leptons. It was found that the latest experimental data for the muon MDM favor a scenario in which the spin-1 unparticle contribution is dominated by the vector couplings since the contribution from axial-vector couplings is negative. It is also found that the decay $\tau \to 3\mu$ can constrain considerably the allowed magnitude of the $\lambda_V^{\mu\mu}$ and $\lambda_V^{\mu\tau}$ couplings.

As far as the EDM is concerned, since it depends on several free parameters, we content ourselves with estimating its order of magnitude. It is found that that the unparticle contributions are well below the experimental limits of the electron and muon EDMs.

Acknowledgments

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Appendix A: Decay $\ell_i^- \to \ell_j^- \ell_k^- \ell_k^+$

In this appendix we present the calculation for the $\ell_i^- \to \ell_j^- \ell_k^- \ell_k^+$ decay mediated by spin-1 unparticles, which proceeds through two tree-level Feynman diagrams (in the first diagram the unparticle couples to the final $\ell_k^- \ell_k^+$ leptons, while the second diagram is obtained through the exchange of $\ell_j^- \leftrightarrow \ell_k^-$). After introducing the Feynman rules for the spin-1 unparticle, the decay width can be written as

$$\Gamma(\ell_i^- \to \ell_j^- \ell_k^- \ell_k^+) = \frac{m_i}{2^9\pi^3}\left|\frac{A_{d_\mathcal{U}}}{\sin(d\pi)}\right|^2 \left(\frac{m_i}{\Lambda_\mathcal{U}}\right)^{4(d_\mathcal{U}-1)} \int dx_1 \int dx_2 \left(|\mathcal{M}_1|^2 + |\mathcal{M}_2|^2 - 2\mathrm{Re}(\mathcal{M}_{12})\right). \tag{A1}$$

If $j = k$ a factor of $1/(2!)$ must be included as there are two identical particles in the final state. The integration area on the x_1 vs. x_2 plane is

$$\begin{aligned} 4s_k^2 - s_j^2 &\leq x_1 \leq 1 - 2s_j, \\ x_2 &\gtreqless \frac{1}{2}\left(1 - x_1 \mp \sqrt{\frac{\left((1-x_1)^2 - 4s_j^2\right)\left(x_1 + s_j^2 - 4s_k^2\right)}{x_1 + s_j^2}}\right) \end{aligned} \tag{A2}$$

where $s_j = m_j/m_i$. In addition, $\mathcal{M}_i$ arises from the squared amplitude corresponding to each Feynman diagram and $\mathcal{M}_{12}$ from their interference. These terms depend on the $d_\mathcal{U}$ dimension and the x_1, x_2 variables, and are given by:

$$|\mathcal{M}_1|^2 = |\lambda_V^{kk}|^2 \Big(|\lambda_V^{ij}|^2 f_1(s_j, s_k) + |\lambda_A^{ij}|^2 f_1(-s_j, s_k)\Big) + |\lambda_A^{kk}|^2 \Big(|\lambda_V^{ij}|^2 f_2(s_j, s_k) + |\lambda_A^{ij}|^2 f_2(-s_j, s_k)\Big), \tag{A3}$$

$$\begin{aligned} |\mathcal{M}_2|^2 &= |\lambda_V^{ik}|^2 \Big(|\lambda_V^{jk}|^2 g_1(s_j, s_k) + |\lambda_A^{jk}|^2 g_1(-s_j, s_k)\Big) + |\lambda_A^{ik}|^2 \Big(|\lambda_V^{jk}|^2 g_2(s_j, s_k) + |\lambda_A^{jk}|^2 g_2(-s_j, s_k)\Big) \\ &+ 2\mathrm{Re}\Big(\Big(\lambda_A^{ik}\lambda_V^{ik}\lambda_V^{*jk}\lambda_A^{*jk}\Big)\Big) g_3(s_j, s_k) \end{aligned} \tag{A4}$$

$$\begin{aligned} \mathcal{M}_{12} &= \lambda_A^{ik}\Big(\lambda_A^{kk}\Big(\lambda_V^{*ij}\lambda_V^{*jk} h_1(s_j, s_k) + \lambda_A^{*ij}\lambda_A^{*jk} h_1(-s_j, s_k)\Big) + \lambda_V^{kk}\Big(\lambda_V^{*jk}\lambda_A^{*ij} h_2(s_j, s_k) + \lambda_A^{*jk}\lambda_V^{*ij} h_2(-s_j, s_k)\Big)\Big) \\ &+ \left(V \leftrightarrow A, h_1(s_j, s_k) \leftrightarrow h_2(-s_j, -s_k)\right), \end{aligned} \tag{A5}$$

For the sake of clarity we omitted the explicit dependence on x_1, x_2 and $d_\mathcal{U}$. The f_i, g_i and h_i functions are

$$f_1(u,v) = -\left(2u^3 + (1+x_1)\,u^2 + 2\left(2v^2 + x_1\right)u + (x_1 - 1)\left(2v^2 + x_1\right) + 2\left(x_1 + x_2 - 1\right)x_2\right)\left(u^2 + x_1\right)^{2(d_\mathcal{U}-2)}, \tag{A6}$$

$$f_2(u,v) \;=\; f_1(u,-v) + 2v^2\,(2u+x_1-1)\,(u\,(3u+2)+2x_1+1)\left(u^2+x_1\right)^{2d_{\mathcal{U}}-5}, \tag{A7}$$

$$\begin{aligned} g_1(u,v) \;=\; & \frac{1}{2}\Big[u^4\,(v-1)\left(x_2\,(v+3)+2v^2+v+1\right)+u^2\Big((5-x_2)\,v^4-((x_2-10)\,x_2+4x_1\,(x_2+1)-3)\,v^2 \\ +\; & 2x_2^2v-2v^5+x_2\left(-2x_2^2+x_2-4x_1\,(x_2+1)+1\right)\Big)-2x_2^3\,(2v\,(v+1)+2x_1-1)-4\,(x_1-1)^2 \\ -\; & 2uv\,(2v+x_2-1)\left(v^2+x_2\right)(v\,(3v+2)+2x_2+1)\,v^4-x_2^2\left((4x_1-3)\,v^2+3v^4+10v^3+4\,(x_1-1)\,x_1\right) \\ -\; & x_2v^2\left(6v^3+v^2+4x_1\,(2x_1-3)+3\right)-2x_2^4\Big]\left(v^2+x_2\right)^{2(d_{\mathcal{U}}-3)}, \end{aligned} \tag{A8}$$

$$g_2(u,v) = g_1(-u,-v), \tag{A9}$$

$$g_3(u,v) = -4\left(u^2-v^2\right)(1+x_2)\left(v^2+x_2\right)^{2(d_{\mathcal{U}}-2)} \tag{A10}$$

$$\begin{aligned} h_1(u,v) \;=\; & \frac{1}{2}\Big[u^6\,(v+1)+u^5\left(1-x_2(v-1)+v^2\right)+u^4\Big((x_1+x_2-4)\,v^2+(x_1-x_2)\,v \\ -\; & 4v^3+2\left(x_2^2+x_1\,(x_2+1)\right)\Big)-u^3\Big(-x_2^2\,(v+1)-x_2\,(v\,(2\,(v-2)\,v-x_1+1)+3x_1) \\ +\; & v\,(v\,(v\,(4v+x_1-1)-2x_1+4)+1)+x_2-x_1\Big)+u^2\Big(-2\,(x_1+1)\,v^4+(-3x_1-2x_2+3)\,v^3 \\ +\; & \left(3x_1^2+(x_2-7)\,x_1+(x_2-7)\,x_2+1\right)v^2-x_2\,(x_1+x_2-1)\,v+2\,(x_2-1)\,x_2^2+2x_1x_2\,(3x_2-1) \\ +\; & x_1^2\,(4x_2+1)\Big)-u\Big(-x_1x_2^2\,(v+1)-x_1x_2\,(v\,(2\,(v-2)\,v+1)+2x_1-1) \\ +\; & v\left(v\left(x_1\,(v\,(4v-3)+2)-v-2x_1^2-1\right)+x_1\right)\Big)-\left(x_1^2-1\right)v^4+(x_1\,(1-2x_2)-1)\,v^3 \\ -\; & x_1\,(x_2-1)\,x_2v+\left(2x_2+x_1\left(2x_1^2+(2x_2-3)\,x_1+(x_2-3)\,x_2+1\right)\right)v^2 \\ +\; & 2x_1x_2\,(x_1+x_2-1)\,(x_1+x_2)\Big]\left(u^2+x_1\right)^{d_{\mathcal{U}}-3}\left(v^2+x_2\right)^{d_{\mathcal{U}}-3}, \end{aligned} \tag{A11}$$

$$\begin{aligned} h_2(u,v) \;=\; & \frac{1}{2}\Big[u^3\,(x_2+1)\,(v-1)+u^2\left((x_1+2x_2-2)\,v^2-(v^2-1)v+x_1+2x_2\,(x_1+x_2)\right) \\ +\; & u\left((2x_1+3x_2-1)\,v^3-2\,(x_1+3x_2-1)\,v^2+(x_2\,(2x_1+3x_2-2)-1)\,v-6v^4-x_2\,(2x_1+x_2-1)\right) \\ +\; & u^4\,(v+1)+x_2^2\,(3\,(v-1)\,v+4x_1-2)+(x_1-1)\,v^2\,(v\,(3v-2)+2x_1-1) \\ +\; & x_2\left((6x_1-4)\,v^2+(1-2x_1)\,v-3v^3+2\,(x_1-1)\,x_1\right)+2x_2^3\Big]\left(u^2+x_1\right)^{d_{\mathcal{U}}-2}\left(v^2+x_2\right)^{d_{\mathcal{U}}-3}. \end{aligned} \tag{A12}$$

1. Decay $\ell_i \to 3l_j$

In the case of this decay, the above expressions simplify considerably. We can write the decay width as

$$\begin{aligned} \Gamma(\ell_i \to 3\ell_j) \;=\; & \frac{m_i}{2^{10}\pi^3}\left|\frac{A_{d_{\mathcal{U}}}}{\sin(d\pi)}\right|^2\left(\frac{m_i}{\Lambda_{\mathcal{U}}}\right)^{4(d_{\mathcal{U}}-1)}\Big[|\lambda_V^{jj}|^2\left(|\lambda_V^{ij}|^2\eta_1(s_j,d_{\mathcal{U}})+|\lambda_A^{ij}|^2\eta_1(-s_j,d_{\mathcal{U}})\right) \\ +\; & |\lambda_A^{jj}|^2\left(|\lambda_V^{ij}|^2\eta_2(s_j,d_{\mathcal{U}})+|\lambda_A^{ij}|^2\eta_2(-s_j,d_{\mathcal{U}})\right)+2\mathrm{Re}\left(\lambda_A^{jj}\lambda_V^{*jj}\lambda_V^{ij}\lambda_A^{*ij}\right)\eta_3(s_j,d_{\mathcal{U}})\Big], \end{aligned} \tag{A13}$$

with

$$\eta_i(z,d_{\mathcal{U}}) = \int dx_1 \int dx_2 \xi_i(z,d_{\mathcal{U}}) \tag{A14}$$

where the integration limits on the x_1 vs. x_2 plane are now given by

$$
\begin{aligned}
3s_j^2 \;&\leq\; x_1 \leq 1-2s_j, \\
x_2 \;&\gtreqless\; \frac{1}{2}\left(1-x_1 \mp \sqrt{\frac{\left((1-x_1)^2-4s_j^2\right)\left(x_1-3s_j^2\right)}{x_1+s_j^2}}\right),
\end{aligned}
\tag{A15}
$$

and the ξ_i functions, whose explicit dependence on x_1 and x_2 has also been omitted for clarity, are given by

$$
\begin{aligned}
\xi_1(z,d_{\mathcal{U}}) \;&=\; -\frac{1}{2}\Big(2q_1^{2(d_{\mathcal{U}}-2)}\left((z+1)(3z-1)x_1+x_1^2+2\left(x_1+x_2-1\right)x_2+(6z-1)z^2\right) \\
&+\; \left(q_1q_2\right)^{d_{\mathcal{U}}-2}\left(x_1+x_2+z-1\right)\left(x_1+x_2+(3z+1)z\right)\Big)+(x_1\leftrightarrow x_2),
\end{aligned}
\tag{A16}
$$

$$
\begin{aligned}
\xi_2(z,d_{\mathcal{U}}) \;&=\; \frac{1}{2}\Big(4q_1^{2d_{\mathcal{U}}-5}\left(z+1\right)^2\left(x_1+2z-1\right)z^2+2q_1^{2d_{\mathcal{U}}-4}\left((z+1)^2x_1-2x_2^2-x_1^2-2\left(x_1-1\right)x_2-(10z+3)z^2\right) \\
&+\; 2q_1^{d_{\mathcal{U}}-3}q_2^{d_{\mathcal{U}}-2}(z+1)\left(x_1+2z-1\right)\left(x_1+2z^2+z\right)z \\
&-\; \left(q_1q_2\right)^{d_{\mathcal{U}}-3}z^2(z+1)^2\left(x_1x_2+(x_1+x_2)z^2+z\left(z\left(z+1\right)-1\right)\right) \\
&-\left(q_1q_2\right)^{d_{\mathcal{U}}-2}\left(\left(x_1+x_2+2\right)z^2+x_1^2+x_2^2+2x_1x_2-x_1-x_2+(7z^2-1)z\right)\Big)+(x_1\leftrightarrow x_2),
\end{aligned}
\tag{A17}
$$

$$
\xi_3(z,d_{\mathcal{U}}) \;=\; -2\Big(q_1^{d_{\mathcal{U}}-3}q_2^{d_{\mathcal{U}}-2}(1+x_1^2-2z^2)z^2+q_1^{d_{\mathcal{U}}-2}q_2^{d_{\mathcal{U}}-2}\left(x_1+x_2-1\right)\left(x_1+x_2\right)\Big)+(x_1\leftrightarrow x_2).
\tag{A18}
$$

We also introduced the notation $q_i = x_i + z^2$.

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